Quantization of free U(1) field

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Covariant quantization

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

naive: $\pi^\mu(x) = \dfrac{\partial \mathcal{L}}{\partial(\partial_0 A_\mu)} \Rightarrow \pi^i = -\pi_i \,, \quad \pi^i = +E^i$

$$\mathcal{L} = -\frac{1}{2}\left(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu\right)$$

$$\pi^i = \frac{\partial}{\partial(\partial_0 A_i)}\left[-\frac{1}{2}\left(\partial_\mu A_\nu \eta^{\mu\alpha}\eta^{\nu\beta}\partial_\alpha A_\beta - \partial_\mu A_\nu \eta^{\nu\alpha}\eta^{\mu\beta}\partial_\alpha A_\beta\right)\right] =$$

$$= -\frac{1}{2}\left(\partial^0 A^i \cdot 2 - \partial^i A^0 \cdot 2\right) = -\dot{A}^i + \partial^i A^0 = -F^{0i} = E^i$$

$$[A^i(t,\bar{x}), \pi^j(t,\bar{y})] \overset{?}{=} -i\delta^{ij}\delta^{(3)}(\bar{x}-\bar{y}) \qquad \frac{\partial \mathcal{L}}{\partial(\partial_0 A^i)} = \pi_i = -\pi^i$$

$$[A^i(t,\bar{x}), A^j(t,\bar{y})] \overset{?}{=} 0$$

Covariant generalization of the above reads: $[A^\mu(t,\bar{x}), \pi^\nu(t,\bar{y})] = i\eta^{\mu\nu}\delta^{(3)}(\bar{x}-\bar{y})$ (*)

$$[A^\mu(t,\bar{x}), A^\nu(t,\bar{y})] = 0$$

(*) can not be correct since $\dfrac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0$, so there is no π^0; $\pi^0 = 0$ so it must commute with A^0

Let's modify $\mathcal{L}$ to fix the problem

$$\mathcal{L} \to \mathcal{L}' = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2 \qquad \text{(no gauge invariance)}$$

Then
$$\mathcal{L}' = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{2}(\partial_\mu A^\mu)^2 = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \underbrace{\frac{1}{2}\partial_\mu A_\nu \partial^\nu A^\mu}_{\downarrow} - \frac{1}{2}(\partial_\mu A^\mu)^2$$

$$-\partial_\mu(\partial^\nu A_\nu)A^\mu + \partial^\nu[(\partial_\mu A_\nu)A^\mu] =$$

$$\mathcal{L}' = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \text{total derivative} \quad \Big\{ \; = (\partial^\nu A_\nu)(\partial_\mu A^\mu) - \underbrace{\partial_\mu[(\partial^\nu A_\nu)A^\mu] + \partial^\nu[(\partial_\mu A_\nu)A^\mu]}_{-\partial_\mu\left[(\partial^\nu A_\nu)A^\mu - (\partial^\nu A^\mu)A_\nu\right]}$$

$$\partial_\mu[(\partial^\nu A^\mu)A_\nu]$$

$$\pi^0 = \frac{\partial \mathcal{L}'}{\partial(\partial_0 A_0)} = -\cancel{\tfrac{1}{2}}\cancel{2}(\partial^0 A^0) \neq 0$$

- Find EoM implied by $\mathcal{L}'$

$$\partial^\mu\left[\frac{\partial\mathcal{L}'}{\partial(\partial^\mu A^\nu)}\right] = -\partial^\mu\left[F_{\mu\nu} + \frac{1}{2}2(\partial_\alpha A^\alpha)\eta_{\mu\nu}\right] = -\partial^\mu(\partial_\mu A_\nu - \cancel{\partial_\nu A_\mu}) - \partial_\nu \cancel{\partial_\mu A^\mu} = -\Box A_\nu = 0$$

$p = (p, 0, 0, p)$

$\omega_p = p$

$$\Box A_\mu = 0 \quad\Rightarrow\quad A_\mu(x) = \int \frac{d^3p}{(2\pi)^3(2\omega_p)^{1/2}} \sum_{\lambda=0}^{3}\left[\epsilon_\mu(p,\lambda)\, a_{p,\lambda}\, e^{-ipx} + \epsilon^*_\mu(p,\lambda)\, a^+_{p,\lambda}\, e^{ipx}\right]$$

$\epsilon^\mu(p,0) = (1,0,0,0) \qquad \epsilon^\mu(p,1) = (0,1,0,0)$

$\epsilon^\mu(p,2) = (0,0,1,0) \qquad \epsilon^\mu(p,3) = (0,0,0,1)$

$\Rightarrow \epsilon^\mu(p,\lambda) = \eta^\mu{}_\lambda$

$\epsilon_\mu(p,\lambda)\cdot\epsilon^\mu(p,\lambda') = \eta_{\lambda\lambda'}$,

$\epsilon_\mu(p,0)\cdot p^\mu \neq 0$, $\quad \epsilon_\mu(p,3)\cdot p^\mu \neq 0$, $\quad \underline{\epsilon_\mu(p,1)\cdot p^\mu = \epsilon_\mu(p,2)\cdot p^\mu = 0}$

$\Downarrow$

$\epsilon^\mu(p,1),\ \epsilon^\mu(p,2)$ – transverse

– Show that $[a_{p,\lambda}, a^+_{p',\lambda'}] = -(2\pi)^3\delta^{(3)}(\bar p - \bar p')\eta_{\lambda\lambda'}$ $\qquad$ $\underline{\eta_{00} = -\eta_{ii} = 1}$

$[a_{p,\lambda}, a_{p',\lambda'}] = [a^+_{p,\lambda}, a^+_{p',\lambda'}] = 0$

imply $\quad [A^\mu(t,\bar x), \Pi^\nu(t,\bar y)] = i\eta^{\mu\nu}\delta^{(3)}(\bar x - \bar y)$

$|\bar p, \lambda\rangle = (2\omega_p)^{1/2} a^+_{p,\lambda}|0\rangle$

– Find $\langle p,\lambda|p,\lambda\rangle$

$$\langle p,\lambda|p,\lambda\rangle = 2\omega_p\langle 0|a_{p,\lambda}a^+_{p,\lambda}|0\rangle = 2\omega_p\langle 0|[a_{p,\lambda}, a^+_{p,\lambda}]|0\rangle = -2\omega_p \underbrace{(2\pi)^3\delta^{(3)}(0)}_{V}\eta_{\lambda\lambda} =$$

the state $|p,0\rangle$ has a negative norm $\Leftarrow$ $\qquad = -2\omega_p V\eta_{\lambda\lambda}$

Note that we have modified the Lagrangian by adding $-\frac{1}{2}(\partial_\alpha A^\alpha)^2$,
now we fix that by requiring that

$$\langle phys'|\partial_\mu A^\mu|phys\rangle = 0 \qquad (*)$$

So, we impose the Lorentz gauge condition on operators acting on physical fields (definition of the subspace of physical states).

- We have to check if the gauge condition $(*)$ eliminate the states with negative norm.

$$\partial_\mu A^\mu = (\partial_\mu A^\mu)^+ + (\partial_\mu A^\mu)^- \quad \text{with} \quad \left.\begin{matrix}(\partial_\mu A^\mu)^+ \\ (\partial_\mu A^\mu)^-\end{matrix}\right\} = \int \frac{d^3p}{(2\pi)^3 (2\omega_p)^{1/2}} \sum_{\lambda=0}^{3} \begin{cases} -i\, \epsilon^\mu(p,\lambda)\, p_\mu\, a_{p,\lambda}\, e^{-ipx} \\ +i\, \epsilon^{\mu *}(p,\lambda)\, p_\mu\, a^+_{p,\lambda}\, e^{ipx} \end{cases}$$

$$\Downarrow$$

$$\langle phys' | \qquad \boxed{(\partial_\mu A^\mu)^+ |phys\rangle = 0} \qquad \Leftarrow \qquad (\partial_\mu A^\mu)^- = \left[(\partial_\mu A^\mu)^+ \right]^+$$

$$\downarrow$$

$$\langle phys' | (\partial_\mu A^\mu)^- = 0 \qquad | \, |phys\rangle$$

$$\downarrow$$

$$\langle phys' | (\partial_\mu A^\mu)^+ + (\partial_\mu A^\mu)^- | phys\rangle = 0$$

Consider a state $|\psi\rangle = \sum_{\lambda'} c_{\lambda'} a^+_{p',\lambda'} |0\rangle \qquad p'^\mu = (p', 0, 0, p')$

For $|\psi\rangle$ to be a physical state we require

$$(\partial_\mu A^\mu)^+ |\psi\rangle = i \int \frac{d^3p}{(2\pi)^3 (2\omega_p)^{1/2}} \sum_{\lambda=0}^{3} \epsilon(p,\lambda)\cdot p \; a_{p,\lambda} \sum_{\lambda'=0}^{3} c_{\lambda'} a^+_{p',\lambda'} |0\rangle e^{-ipx} = 0$$

$$= \;- \,''\, - \sum_{\lambda,\lambda'=0}^{3} \epsilon(p,\lambda)\cdot p \; c_{\lambda'} \Big(\underbrace{[a_{p,\lambda}, a^+_{p',\lambda'}]}_{= -(2\pi)^3 \delta^{(3)}(\bar{p}-\bar{p}')\eta_{\lambda\lambda'}} + a^+_{p',\lambda'} a_{p,\lambda} \Big) |0\rangle e^{-ipx} =$$

$$= \frac{-i}{(2\pi)^3 (2\omega_{p'})^{1/2}} \left[-(2\pi)^3\right] \Big[\epsilon(p',0)\cdot p' c_0 - \underbrace{\epsilon(p',3)\cdot p'}_{-\epsilon(p',0)\cdot p'} c_3 \Big] e^{-ip'x} |0\rangle = 0$$

$$\Downarrow$$

$$\boxed{c_0 + c_3 = 0}$$

$$\Downarrow$$

$$\begin{cases} \epsilon^\mu(p,0) = (1,0,0,0) \qquad \epsilon^\mu(p,3) = (0,0,0,1) \\ p^\mu = (p, 0, 0, p) \end{cases}$$

$$\downarrow \; 1$$

$$= (p, 0, 0, p)$$

The most general 1-particle physical state : $\underbrace{|\psi_T\rangle}_{\text{created by } a^+_{p,1} \text{ or } a^+_{p,2}} + c|\phi\rangle$ for $|\phi\rangle = \left(a^+(p,0) - a^+(p,3)\right)|0\rangle$

— Show that $\langle\phi|\phi\rangle = 0$ and that

$|\text{phys}\rangle + c|\phi\rangle$ does not change any product

$$\langle\phi|\phi\rangle = \langle 0|\left(a(p,0) - a(p,3)\right)\left(a^+(p,0) - a^+(p,3)\right)|0\rangle = \langle 0|\Big(a(p,0)\,a^+(p,0) + a(p,3)\,a^+(p,3) +$$
$$- a(p,0)\,a^+(p,3) - a(p,3)\,a^+(p,0)\Big)|0\rangle$$

$$[a(p,\lambda), a^+(p',\lambda')] = -(2\pi)^3\,\delta^{(3)}(\bar p - \bar p')\,\eta_{\lambda\lambda'} \Rightarrow [a(p,0), a^+(p,3)] = 0$$

$$= \langle 0|\left(a(p,0)\,a^+(p,0) + a(p,3)\,a^+(p,3)\right)|0\rangle = \langle 0|\Big([a(p,0), a^+(p,0)] + [a(p,3), a^+(p,3)]\Big)|0\rangle =$$
$$= 0 \quad \text{since} \quad [a(p,0), a^+(p,0)] = -[a(p,3), a^+(p,3)]$$

$$\left(\langle\psi'_T| + c'\langle\phi|\right)\left(|\psi_T\rangle + c|\phi\rangle\right) = \langle\psi'_T|\psi_T\rangle + c\langle\psi'_T|\phi\rangle + c'\langle\phi|\psi_T\rangle +$$
$$+ cc'\,\langle\phi|\phi\rangle = \langle\psi'_T|\psi_T\rangle$$

$$\langle\psi'_T|\phi\rangle \propto \langle 0|\left(c_1^*\,a(p,1) + c_2^*\,a_2(p,2)\right)\left(a^+(p,0) - a^+(p,3)\right)|0\rangle = 0$$

Therefore $|\psi_T\rangle + c|\phi\rangle$ has the same norm and products as $|\psi_T\rangle$

where $|\psi_T\rangle \propto \left(c_1\,a^+(p,1) + c_2\,a^+(p,2)\right)|0\rangle$

Hamiltonian

$$\mathcal{L} = -\frac{1}{2}(\partial_\alpha A_\beta)(\partial^\alpha A^\beta) \Rightarrow \Pi^\mu \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 A_\mu)} = -\partial^0 A^\mu = -\dot{A}^\mu$$

$$T^{\mu\nu} = -\eta^{\mu\nu}\mathcal{L} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\alpha)} \cdot \partial^\nu A_\alpha \Rightarrow \mathcal{H} = T^{00} = -\mathcal{L} + \Pi^\alpha \dot{A}_\alpha = -\mathcal{L} - \dot{A}_\alpha \dot{A}^\alpha =$$

$$= \frac{1}{2}(\partial_\alpha A_\beta)(\partial^\alpha A^\beta) - \dot{A}_\alpha \dot{A}^\alpha = \frac{1}{2}\dot{A}_\beta \dot{A}^\beta + \frac{1}{2}\partial_i A_\beta \partial^i A^\beta - \dot{A}_\alpha \dot{A}^\alpha =$$

$$= -\frac{1}{2}\dot{A}_\alpha \dot{A}^\alpha - \frac{1}{2}(\partial_i A_\alpha)(\partial_i A^\alpha) =$$

$$= -\frac{1}{2}\left[(\dot{A}_0)^2 - (\dot{A}_k)^2\right] - \frac{1}{2}\left[(\partial_i A_0)^2 - (\partial_i A_k)^2\right] =$$

$$= \frac{1}{2}\left[(\dot{A}_k)^2 + (\bar{\nabla} A_k)^2\right] - \frac{1}{2}\left[(\dot{A}_0)^2 + (\bar{\nabla} A_0)^2\right]$$

wrong sign!

$$H = \int d^3x\, \mathcal{H} = -\frac{1}{2}\int d^3x \left(\dot{A}_\mu \dot{A}^\mu + \partial_i A_\mu \partial_i A^\mu\right):$$

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3 (2\omega_p)^{1/2}} \sum_{\lambda=0}^{3} \left[\epsilon_\mu(p,\lambda)\, a_{p,\lambda}\, e^{-ipx} + \epsilon^*_\mu(p,\lambda)\, a^+_{p,\lambda}\, e^{ipx}\right]$$

$$\underbrace{\dot{A}_\mu(x)} = \int \frac{d^3p}{(2\pi)^3 (2\omega_p)^{1/2}} (-i\omega_p) \sum_{\lambda=0}^{3} \left[\epsilon_\mu(p,\lambda)\, a_{p,\lambda}\, e^{-ipx} - \epsilon^*_\mu(p,\lambda)\, a^+_{p,\lambda}\, e^{ipx}\right]$$

$$-\frac{1}{2}\int d^3x\, \dot{A}_\mu \dot{A}^\mu = -\frac{1}{2}\int d^3x \int \frac{d^3p}{(2\pi)^3(2\omega_p)^{1/2}} \int \frac{d^3p'}{(2\pi)^3(2\omega_{p'})^{1/2}} (-i)^2 \omega_p \omega_{p'} \sum_{\lambda,\lambda'} \left[\epsilon_\mu(p,\lambda)\, a_{p,\lambda}\, e^{-ipx} - \epsilon^*_\mu(p,\lambda)\, a^+_{p,\lambda}\, e^{ipx}\right]\left[\epsilon^\mu(p',\lambda')\, a_{p',\lambda'}\, e^{-ip'x} - \epsilon^{\mu *}(p',\lambda')\, a^+_{p',\lambda'}\, e^{ip'x}\right] =$$

$$= +\frac{1}{2}\int \frac{d^3p}{(2\pi)^3(2\omega_p)^{1/2}} \frac{1}{\cancel{(2\pi)^3}(2\omega_p)^{1/2}} \cancel{(2\pi)^3}\, \omega_p^2 \sum_{\lambda,\lambda'} \Big\{ -a_{p,\lambda}\, a^+_{p,\lambda'}\, \epsilon_\mu(p,\lambda)\cdot\epsilon^{\mu *}(p,\lambda') - a^+_{p,\lambda}\, a_{p,\lambda'}\, \epsilon^*_\mu(p,\lambda)\epsilon^\mu(p,\lambda') +$$

$$\boxed{\int d^3x\, e^{-i(\bar{p}\pm\bar{p}')x}}$$

$$+ a_{p,\lambda}\, a_{-p,\lambda'}\, \epsilon_\mu(p,\lambda)\cdot\epsilon^\mu(-p,\lambda')\, e^{-2i\omega_p t} + a^+_{p,\lambda}\, a^+_{-p,\lambda'}\, \epsilon^*_\mu(p,\lambda)\cdot\epsilon^{\mu *}(-p,\lambda')\, e^{2i\omega_p t}\Big\}$$

$$\underbrace{\partial_i A_\mu(x)} = \int \frac{d^3p}{(2\pi)^3 (2\omega_p)^{1/2}} (ip^i) \sum_{\lambda=0}^{3} \left[\epsilon_\mu(p,\lambda)\, a_{p,\lambda}\, e^{-ipx} - \epsilon^*_\mu(p,\lambda)\, a^+_{p,\lambda}\, e^{ipx}\right]$$

$$-\frac{1}{2}\int d^3x\, \partial_i A_\mu \partial_i A^\mu = -\frac{1}{2}\int d^3x \int \frac{d^3p}{(2\pi)^3(2\omega_p)^{1/2}} \int \frac{d^3p'}{(2\pi)^3(2\omega_{p'})^{1/2}} \sum_{\lambda,\lambda'} (i\bar{p}) \left[\epsilon_\mu(p,\lambda) a_{p,\lambda} e^{-ipx} - \epsilon^*_\mu(p,\lambda) a^+_{p,\lambda} e^{ipx}\right] (i\bar{p}') \left[\epsilon^\mu(p',\lambda') a_{p',\lambda'} e^{-ip'x} - \epsilon^{\mu *}(p',\lambda') a^+_{p',\lambda'} e^{ip'x}\right] =$$

$$= +\frac{1}{2}\int \frac{d^3p}{(2\pi)^3(2\omega_p)^{1/2}} \frac{1}{\cancel{(2\pi)^3}(2\omega_p)^{1/2}} \cancel{(2\pi)^3}\, \bar{p}^2 \sum_{\lambda,\lambda'} \Big\{ -a_{p,\lambda} a^+_{p,\lambda'} \epsilon_\mu(p,\lambda)\epsilon^{\mu *}(p,\lambda') - a^+_{p,\lambda} a_{p,\lambda'} \epsilon^*_\mu(p,\lambda)\cdot\epsilon^\mu(p,\lambda') +$$

$$\int d^3x\, e^{-i(\bar{p}\pm\bar{p}')x}$$

$$- a_{p,\lambda} a_{-p,\lambda'} \epsilon_\mu(p,\lambda)\cdot\epsilon^\mu(-p,\lambda') e^{-2i\omega_p t} - a^+_{p,\lambda} a^+_{-p,\lambda'} \epsilon^*_\mu(p,\lambda)\cdot\epsilon^{\mu *}(-p,\lambda') e^{2i\omega_p t} \Big\}$$

$$H = -\frac{1}{2}\int d^3x \left(\dot{A}_\mu \dot{A}^\mu + \partial_i A_\mu \partial_i A^\mu\right) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{4\omega_p} \sum_{\lambda,\lambda'=0}^{3} : \Big\{ a_{p,\lambda} a^+_{p,\lambda'} \underbrace{\epsilon_\mu(p,\lambda)\epsilon^{\mu *}(p,\lambda')}_{\eta^{\lambda\lambda'}} \underbrace{(-\omega_p^2 - \bar{p}^2)}_{-2\omega_p^2} + a^+_{p,\lambda} a_{p,\lambda'} \epsilon^*_\mu(p,\lambda)\epsilon^\mu(p,\lambda') \underbrace{(-\omega_p^2 - \bar{p}^2)}_{-2\omega_p^2} +$$

$$+ a_{p,\lambda} a_{-p,\lambda'} \epsilon_\mu(p,\lambda)\cdot\epsilon^\mu(-p,\lambda') e^{-2i\omega_p t} \underbrace{(\omega_p^2 - \bar{p}^2)}_{=0} +$$

$$+ a^+_{p,\lambda} a^+_{-p,\lambda'} \epsilon^*_\mu(p,\lambda)\cdot\epsilon^{\mu *}(-p,\lambda') e^{2i\omega_p t} \underbrace{(\omega_p^2 - \bar{p}^2)}_{\to 0} \Big\} :$$

$$H = -\int \frac{d^3p}{(2\pi)^3} \frac{\omega_p}{2} \sum_{\lambda=0}^{3} \zeta_\lambda : \left(a_{p,\lambda} a^+_{p,\lambda} + a^+_{p,\lambda} a_{p,\lambda}\right) : = \int \frac{d^3p}{(2\pi)^3} \omega_p \left(-a^+_{p,0} a_{p,0} + \sum_{i=1}^{3} a^+_{p,i} a_{p,i}\right)$$

$$\zeta_\lambda = \begin{cases} +1 & \text{for } \lambda = 0 \\ -1 & \text{for } \lambda = 1,2,3 \end{cases}$$

$$\bar{P} = \int \frac{d^3p'}{(2\pi)^3} \bar{p}' \underbrace{\left(-a^+_{p',0} a_{p',0} + \sum_{\lambda=1,2,3} a^+_{p',\lambda} a_{p',\lambda}\right)}_{-\eta^{\lambda\lambda'} a^+_{p,\lambda} a_{p,\lambda'}}$$

– Show that the condition $c_0 + c_3 = 0$ can be rewritten as

$$\left(a_{p,0} - a_{p,3}\right)|\psi\rangle = 0 \qquad (**)$$

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$$(a_{p,0} - a_{p,3})\sum_{\lambda=0}^{3} c_\lambda a^+_{q,\lambda}|0\rangle = \sum_{\lambda=0}^{3} c_\lambda \left(a_{p,0} a^+_{q,\lambda}|0\rangle - a_{p,3} a^+_{q,\lambda}|0\rangle\right) =$$

$$\left[-(2\pi)^3 \delta^{(3)}(\bar{p}-\bar{q}) \underbrace{\eta_{0\lambda}}_{+1} + (2\pi)^3 \delta^{(3)}(\bar{p}-\bar{q}) \underbrace{\eta_{3\lambda}}_{-1}\right]|0\rangle$$

$$= -(2\pi)^3 \delta^{(3)}(\bar{p}-\bar{q})(c_0 + c_3)|0\rangle = 0$$

$$\Downarrow$$

$$c_0 + c_3 = 0$$

– Show that the contributions $-a^+_{p,0} a_{p,0} + a^+_{p,3} a_{p,3}$ cancel each other while acting on physical states

$$\langle phys'| - a^+_{p,0} a_{p,0} + a^+_{p,3} a_{p,3} |phys\rangle \underset{(**)}{=} \langle phys'|(-a^+_{p,0} + a^+_{p,3}) a_{p,0}|phys\rangle \underset{(**)^+}{=} 0$$

$$\Downarrow$$

$$\boxed{\text{only the transverse states } |\psi_T\rangle \text{ generated by } a^+_{p,1}, a^+_{p,2} \text{ contribute to expectation values of } H, \bar{P}}$$

$$\Downarrow$$

$|\psi_T\rangle$ and $|\psi_T\rangle + c|\phi\rangle$ have the same energy and momentum, and the same product with all other physical states

$L = (a^+_{p,0} - a^+_{p,3})|0\rangle$

$$\Downarrow$$

free physical photons have two degrees of freedom (transverse)